Exercise 43

(a) For $f(x) = \dfrac{x}{\ln x}$ find each of the following limits.

$$\text{(i)} \quad \lim_{x \to 0^+} f(x) \qquad \text{(ii)} \quad \lim_{x \to 1^-} f(x) \qquad \text{(iii)} \quad \lim_{x \to 1^+} f(x)$$

(b) Use a table of values to estimate $\lim\limits_{x \to \infty} f(x)$.

(c) Use the information from parts (a) and (b) to make a rough sketch of the graph of f.

Solution

Evaluate each of the limits by plugging in the numbers.

$$\text{(i)} \quad \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{x}{\ln x} = \frac{0}{-\infty} = 0$$

$$\text{(ii)} \quad \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{x}{\ln x} = \frac{1}{-0} = -\infty$$

$$\text{(iii)} \quad \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{x}{\ln x} = \frac{1}{+0} = +\infty$$

Make a table with large values of x to see what happens as $x \to \infty$.

x	$f(x)$
10	4.34294
100	21.7147
1000	144.765
10 000	1085.74
1 000 000	72382.4

The function seems to be unbounded as x increases, so

$$\lim_{x \to \infty} f(x) = \infty.$$